



An Investigation Through Stochastic Procedures for Solving the Fractional Order Computer Virus Propagation Mathematical Model with Kill Signals

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Accepted: 4 July 2022

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Abstract

In this study, the numerical investigations through the stochastic procedures for solving a class of fractional order (FO) computer virus propagation (CVP) mathematical model with kill signals (KS), i.e., CVP-KS is presented. The KS gets alert about those viruses, which can be infected through the computer system to decrease the virus propagation danger. The mathematical model of the CVP-KS is based on the SEIR-KS model. The focus of these investigations is to present the numerical solutions of the FO-SEIR-KS model using the sense of Levenberg–Marquardt backpropagation scheme (LMBS) together with the neural networks (NNs), i.e., LMBS-NNs. The use of the one dynamic of the other makes the model nonlinear. Three different FO values have been used to check the performances of the designed scheme for this FO-SEIR-KS nonlinear mathematical model. The statics used in this study is 80%, 10% and 10% for training, testing and certification for solving the FO-SEIR-KS nonlinear mathematical model. The numerical simulations are performed through the stochastic LMBS-NNs scheme for solving the FO-SEIR-KS nonlinear mathematical model. The obtained results will be compared with the design of database reference solutions based on the Adams–Bashforth–Moulton. In order to accomplish the validity, capability, consistency, competence and accuracy of the LMBS-NNs, the numerical results using the error histograms, regression, mean square error, state transitions and correlation have been provided.

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Keywords Computer virus · Kill signals · Fractional order · Levenberg–Marquardt backpropagation · Adams–Bashforth–Moulton

1 Introduction

The computer-based viruses discovered at the end of last century through the malignant codes with virus, trojan horse, worm, etc. These viruses damaged the small substances, which have been conducted from the computer network without the system's information. At the start, the spreading of the computer viruses was low that was not too harmful, strong and dangerous. The computer viruses become more harmful with the passage of time due to the increase of development and globalization of communication. The spread of these viruses is the threat for the information civilization with the acceleration of their unpleasant activities, like bank accounts, stealing passwords, email address, big financial damages, changing data and breaking the precise procedure of a device [1–3].

In order to check the relation between the computer and real biological viruses, e.g., SLBS [4, 5], SIS [6], SEIRS [7], SIRS [8–10] and stochastic systems [11–13], assisted with the computer networks and virus spread in an appropriate mode. Virus control [14, 15] and virus immunization [16, 17] are two variants of the antivirus systems that have been used to fight against the computer virus spread. Kill Signals (KS), which is known as a warning network for the possible virus contagions and an advanced antivirus apparatus have been presented by Kephart et al. [18].

The focus of fractional calculus is to generalize the differential form of the operators to the non-integer form [19–21]. Recently, fractional calculus has a significant role in numerous areas of mathematics, physics, finance, biology and neural networks [22–25]. On the other hand, the derivative form of the integer can be stated by a fractional order (FO), which can be calculated as a substantial operator to model the various real values with memory. The FO and integer forms of the derivatives are different with the use of nonlocal operator. To analyze the FO derivative, it is considered for the starting to desired time ($t = t_0$ to $t = t_1$). Consequently, the mathematical systems using the real values of the FO derivatives have attracted many scientists of different areas [26]. Moreover, it is observed that the FO derivatives in biology are implemented to model and analyze the rheological cell's features [27]. Likewise, the FO is applied to the electrical cell based on the biological organism [28]. In the mechanical field, the FO derivatives show the complex performance of several physical systems [29, 30]. Few related studies have been presented to solve the FO models is presented in the references [31–40].

To consider the importance of the above-mentioned FO models, the authors are interested in solving the FO computer virus propagation (CVP) system using the kill signals known as SEIR-KS model. At the start, the SEIR-KS model was proposed with the use of classical derivative [41]. The model was divided into five classes: Susceptible $S(x)$, the state that is not infected and virus free; Exposed $E(x)$, infected individuals that makes the virus inactive; Infected $I(x)$, abruptly spreading virus and infected; Recovered $R(x)$, gained immunization and recovered from the virus; KS, i.e., Kill signals are represented by $K(x)$ known as a special class based on the anti-virus epidemic type. After getting the KS successfully, an infected class may attempt to pass its successor, whereas a vulnerable class obtains insusceptibility as KS comprises the cure. As the fourth-class dynamics may be dissociated with the supplementary classes, R component gets lost and SEIR-KS system provides the integer order differential system, which is given as [42]:

$$\begin{cases} \frac{d}{dx} S(x) = bp - \phi S(x)K(x) - \beta S(x)E(x) - \mu S(x), & S_0 = l_1, \\ \frac{d}{dx} E(x) = bq - \chi E(x) + \beta S(x)E(x) - \mu E(x) - \bar{\alpha} E(x), & E_0 = l_2, \\ \frac{d}{dx} I(x) = \bar{\alpha} E(x) - \gamma I(x) - \lambda I(x)K(x) - \mu I(x) - \varepsilon I(x), & I_0 = l_3, \\ \frac{d}{dx} K(x) = \lambda I(x)K(x) - \mu K(x) + \gamma I(x) + \chi E(x), & K_0 = l_4. \end{cases} \quad (1)$$

where l_1, l_2, l_3 and l_4 indicate the initial conditions (ICs) of the above system. At the time x , the SEIR-KS system parameters can be defined as: the exterior system classes are committed to the system at b ratio and p represents a computer's fraction in the system of susceptible, whereas q shows the computers in the exposed network and $q + p = 1$. Due to the crashing system or poor network, each computer linked to the system is separated from the system at μ .

Subsequently, the exposed class has contagion, so each susceptible class can be infected, whereas linking and corresponding with exposed individual at time " x " and deviations into the exposed class with per unit probability time $\beta E(x)$ as β is a constant. The changes of exposed into the infected performed with ratio $\bar{\alpha}$. Due to the implementation of the anti-virus, an infected class become inoculation state with ε ratio. A KS is calculated through exposed to infected class with ratios χ and γ . Each infected and susceptible class places again the KS with λ and ϕ . KS gets slow with the ratio μ due to the drop-out or rebound around the network.

In this study, the numerical solutions of the time fractional order SEIR-KS system have been presented. The general form of the fractional order SEIR-KS system is given as [43]:

$$\begin{cases} D^\alpha S(x) = bp - \phi S(x)K(x) - \beta S(x)E(x) - \mu S(x), & S_0 = l_1, \\ D^\alpha E(x) = bq - \chi E(x) + \beta S(x)E(x) - \mu E(x) - \bar{\alpha} E(x), & E_0 = l_2, \\ D^\alpha I(x) = \bar{\alpha} E(x) - \gamma I(x) - \lambda I(x)K(x) - \mu I(x) - \varepsilon I(x), & I_0 = l_3, \\ D^\alpha K(x) = \lambda I(x)K(x) - \mu K(x) + \gamma I(x) + \chi E(x), & K_0 = l_4, \end{cases} \quad (2)$$

where fractional order operator α is taken as in the sense of Caputo derivative to analyze the dynamics of fractional order SEIR-KS system. Moreover, the value of α is taken between 0 and 1 to analysis the behavior of the nonlinear FO-CVP system using the kill signals as SEIR-KS model to study the behavior for superfast transients as well as superslow evolution which are difficult/seldom to observe by integer order counterpart models. Further details of mathematical model and justification with the help of theoretical proves of the FO-CVP model in (2) are given in the reported study [43].

The aim of this study is to solve the above nonlinear system using the Levenberg—Marquardt backpropagation scheme (LMBS) together with the neural networks (NNs), i.e., LMBS-NNs. The stochastic LMBS-NNs procedures have never implemented before to present the numerical performances of the FO-SEIR-KS mathematical model. The statics used in this study is 80%, 10% and 10% for training, testing and certification for solving the FO-SEIR-KS nonlinear mathematical model. The numerical stochastic LMBS-NNs procedures have the ability and competence to solve the complex, nonlinear, singular, economical, functional, biological and fluid models [44–55]. These stochastic, computing, numerical procedures motivated the authors to solve the FO derivatives of the SEIR-KS nonlinear mathematical model. Few novel features for solving the FO-SEIR-KS mathematical model using the LMBS-NNs are presented as:

- A mathematical fractional order CVP model with KS is successfully solved by applying the stochastic procedures.
- A design of the LMBS along with the artificial NNs is presented first time for solving the mathematical fractional order CVP model with KS.
- Three different FO variations have been provided to solve the mathematical fractional order CVP model with KS.
- The accuracy of the stochastic LMBS is achieved to check the comparison of the obtained results with the Adams–Bashforth–Moulton numerical scheme.
- The obtained absolute error (AE) with the overlapping of 6 to 8 order authenticates the exactness of the designed LMBS-NNs to solve the mathematical fractional order CVP model with KS.
- The presentations of the regression, EHs, correlation, MSE and STs approve the consistency and reliability of the designed LMBS-NNs to solve the mathematical fractional order CVP model with KS.

The remaining paper is proceeded as: The LMBS-NNs is provided in Sect. 2. The simulations of the FO-SEIR-KS using the LMBS-NNs are given in Sect. 3. The conclusions are reported in the final Sect.

2 Methodology: LMBS-NNs

This part of the paper shows the designed LMBS-NNs structure for the mathematical fractional order CVP model with KS. The stochastic LMBS-NNs procedure is classified in two phases. In the first step, essential performances using the stochastic LMBS-NNs are provided, whereas the implementation steps are given to solve the FO-SEIR-KS mathematical system. An appropriate optimization scheme based LMBS-NNs is illustrated in Fig. 1 that shows the multi-layer neurons. Figure 2 represents a single neuron structure. The statics used in this study is 80%, 10% and 10% for training, testing and certification for solving the FO-SEIR-KS nonlinear mathematical model.

3 Numerical Performances of FO-SEIR-KS Model

In this portion, the numerical demonstrations of the FO-SEIR-KS mathematical model is provided by applying the LMBS-NNs. The parameter values to solve FO-SEIR-KS mathematical model are $\bar{\alpha} = 0.4$, $p = 1$, $b = 0.005$, $\phi = 0.01$, $\beta = 0.85$, $\mu = 0.03$, $\chi = 0.19$, $\gamma = 0.06$, $\lambda = 0.12$, $\varepsilon = 0.52$, $l_1 = 0.6$, $l_2 = 0.4$, $l_3 = 0.8$ and $l_4 = 0.6$. Three different FO variations, i.e., $\alpha = 0.5, 0.7$ and 0.9 have been presented in this research study. The comparison of the outcomes is performed for each dynamics of the FO-SEIR-KS by using 13 numbers of neurons. The statics used in this study is 80%, 10% and 10% for training, testing and certification for solving the FO-SEIR-KS nonlinear mathematical model. The obtained solutions by using 13 numbers of neurons to solve the FO-SEIR-KS model are drawn in Fig. 3.

The graphic illustrations to solve the FO-SEIR-KS model using the LMBS-NNs are drawn in Figs. 4, 5, 6, 7 and 8. The STs and MSE measures are illustrated to solve the FO-SEIR-NNs in Fig. 4. The plots of the best curve, training, authentication and testing using the MSE are shown in Fig. 4a–c. The STs best presentation values to solve the FO-SEIR-NNs are demonstrated in Fig. 4d–f at epochs 102, 52 and 42. The best achieved values have been

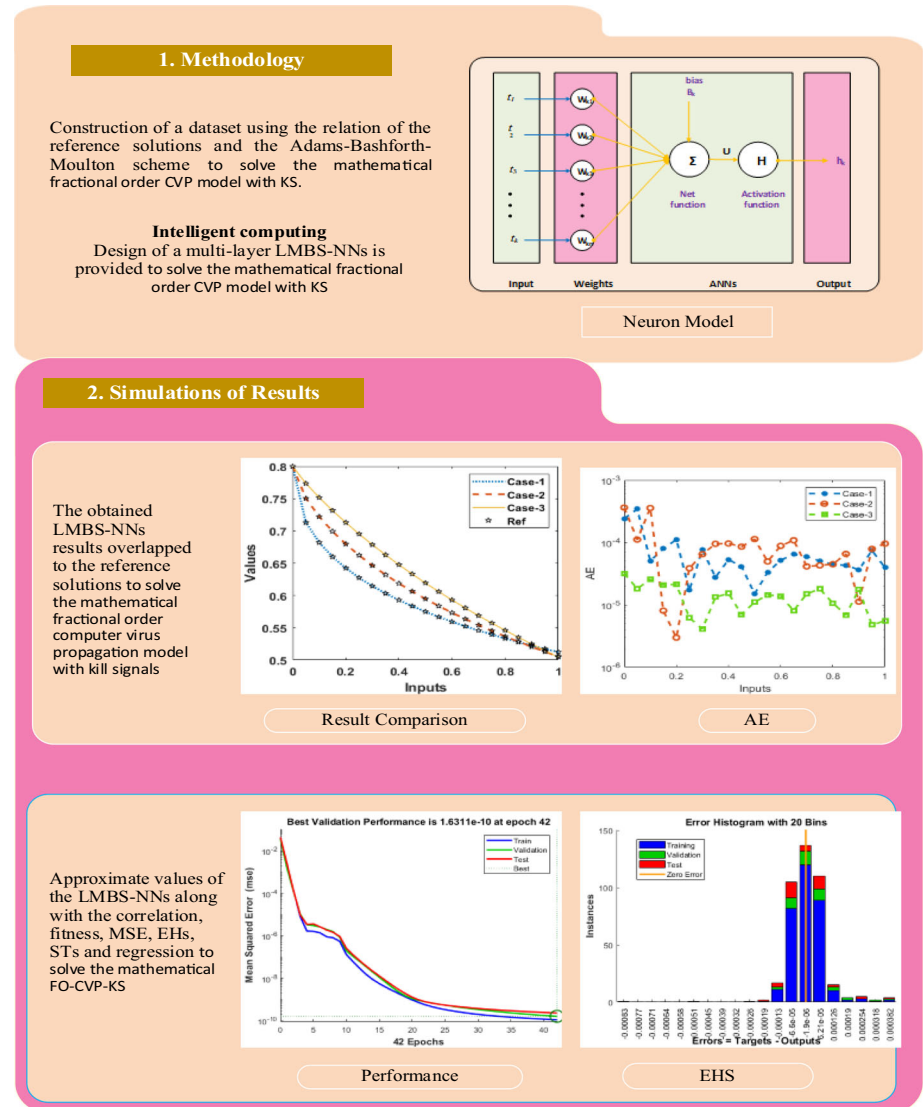


Fig. 1 Workflow illustrations of the LMBS-NNs to solve the mathematical fractional order CVP model with KS

observed at 1.7784×10^{-09} , 1.1833×10^{-08} and 1.6311×10^{-10} . The gradient measures through the LMBS-NNs for the FO-SEIR-NNs mathematical model are observed 9.716×10^{-08} , 9.575×10^{-08} and 9.5447×10^{-08} . The obtained performances represent the precision and convergence of the LMBS-NNs for solving the FO-SEIR-KS mathematical model. The fitting curve plots have been exemplified in Fig. 5 for the FO-SEIR-KS mathematical model using the proposed LMBNNs. These values have been drawn using the comparison presentations of the obtained results through LMBS-NNs. Figure 5d–f is illustrated based on the EHS values, which are calculated 2.79×10^{-05} , 1.96×10^{-06} and 2.16×10^{-07} for case 1, 2 and

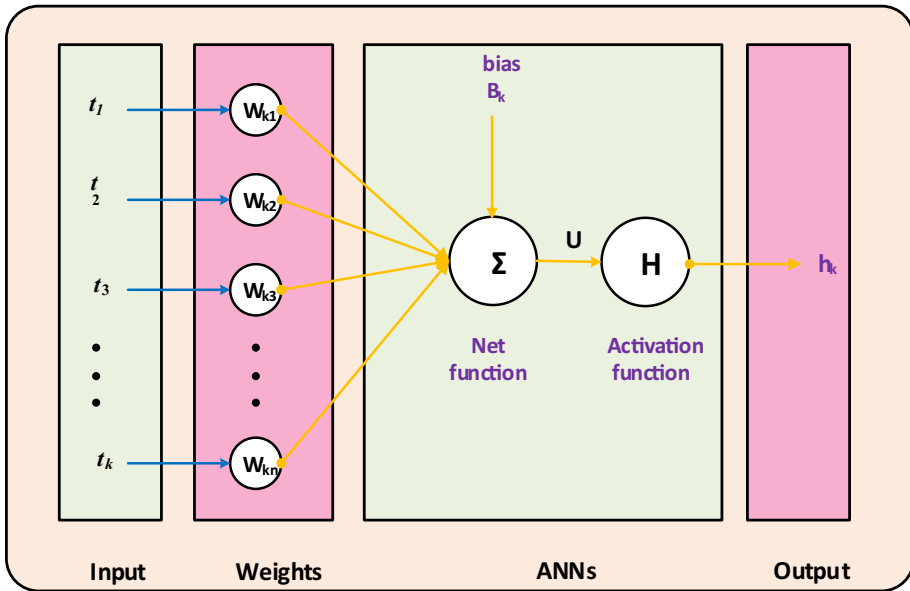


Fig. 2 Single neuron structure

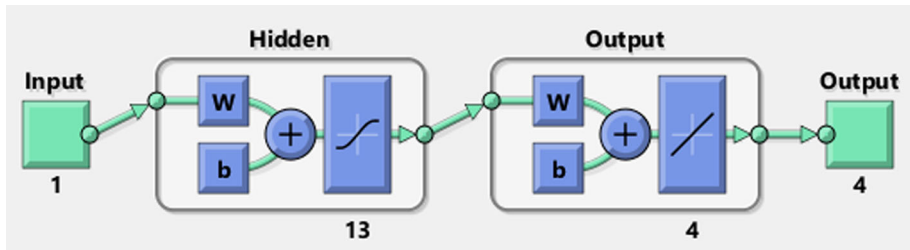


Fig. 3 Construction of the LMBS-NNs for the FO-SEIR-KS model using 13 numbers of neurons

3. The regression performances are illustrated in Figs. 6. These regression measures validate the performances of the perfect model, which are calculated 1. The verification, training and testing values indicate the precision of the LMBS-NNs for the FO-SEIR-NNs. Moreover, the MSE convergence through the complexity, training, epochs, confirmation, testing and backpropagation is provided in Table 1 for the FO-SEIR-KS mathematical model.

The AE illustrations are provided in Figs. 7 and 8, which have been calculated using the comparisons of the FO-HBV-DIS mathematical model. The numerical measures are illustrated for each category of the FO-SEIR-KS mathematical model using the designed LMBS-NNs. The numerical values are shown in Figs. 7, that represents the overlapping of the designed and the reference outcomes. These overlapping signifies the correctness and accuracy of the proposed LMBS-NNs for nonlinear FO-SEIR-KS mathematical model. The AE for each class of the FO-SEIR-KS mathematical model is illustrated in Fig. 8. The AE performances for the susceptible $S(x)$ category lie 10^{-04} to 10^{-05} , 10^{-04} to 10^{-06} and 10^{-05} to 10^{-06} for case 1, 2 and 3 to solve the FO-SEIR-KS nonlinear mathematical model. The

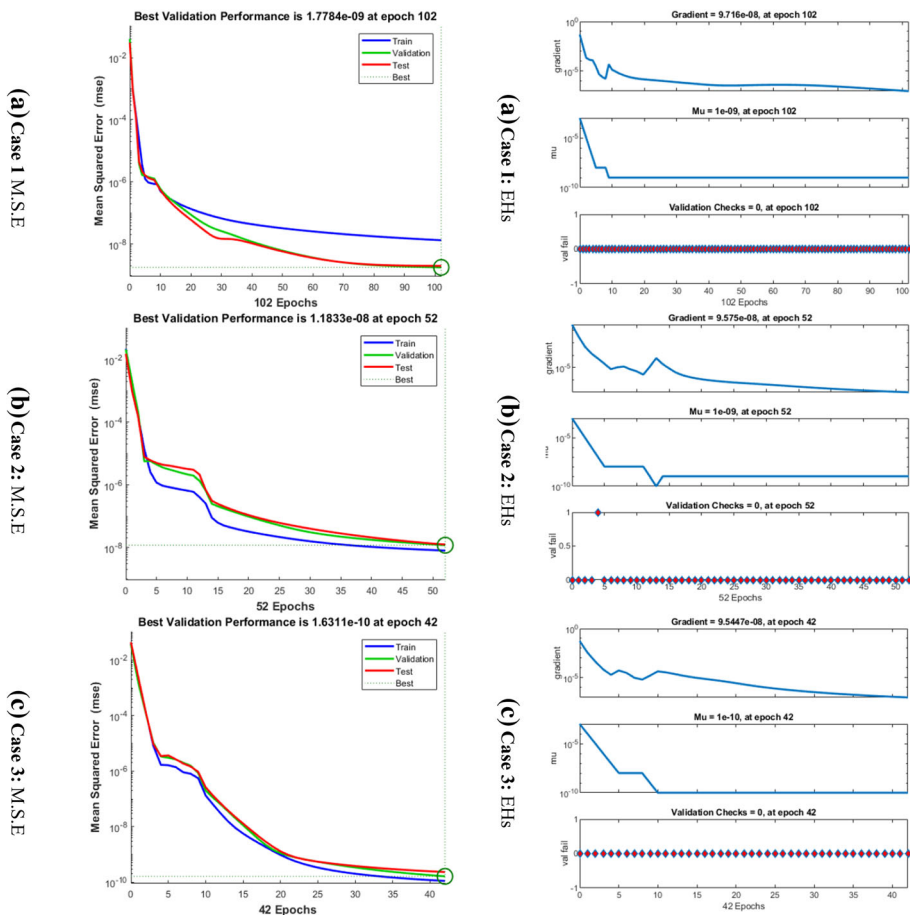


Fig. 4 MSE and STs performances to solve the FO-SEIR-KS mathematical model

AE values for the exposed $E(x)$ lie 10^{-04} to 10^{-05} , 10^{-04} to 10^{-06} and 10^{-05} to 10^{-07} for case 1, 2 and 3 to solve the FO-SEIR-KS nonlinear mathematical model. The AE values for the infected $I(x)$ lie 10^{-04} to 10^{-05} , 10^{-03} to 10^{-05} and 10^{-04} to 10^{-06} for case 1, 2 and 3 to solve the FO-SEIR-KS nonlinear mathematical model. The values of AE for the KS category $K(x)$ lie 10^{-04} to 10^{-06} , 10^{-04} to 10^{-05} and 10^{-05} to 10^{-06} cases 1 to 3 for the FO-SEIR-KS nonlinear mathematical model. The AE values enhance the exactness of the stochastic approach for the FO-SEIR-KS nonlinear mathematical model.

4 Conclusions

In this study, an investigation is presented to solve a fractional order CVP mathematical model with KS using the stochastic procedures of the LMBS-NNs. The fractional order SEIR-KS mathematical nonlinear model is executed to solve three different variants of the fractional order. The statics used in this study are 80%, 10% and 10% for training, testing and

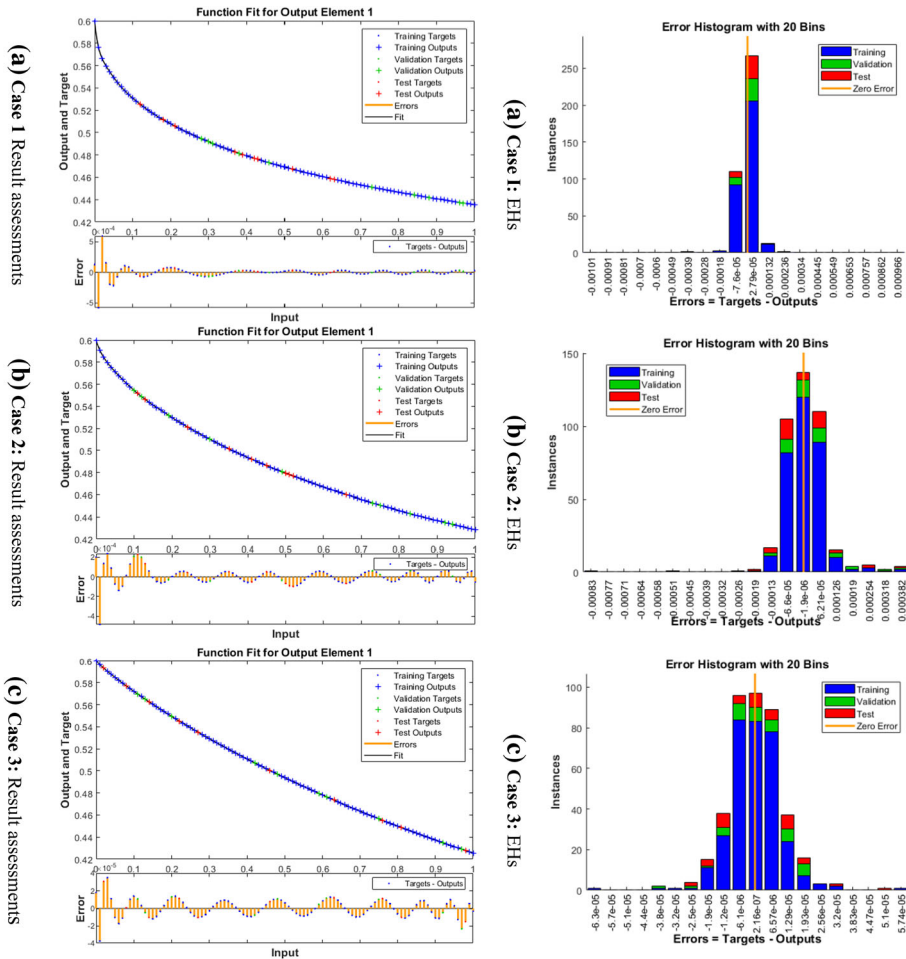
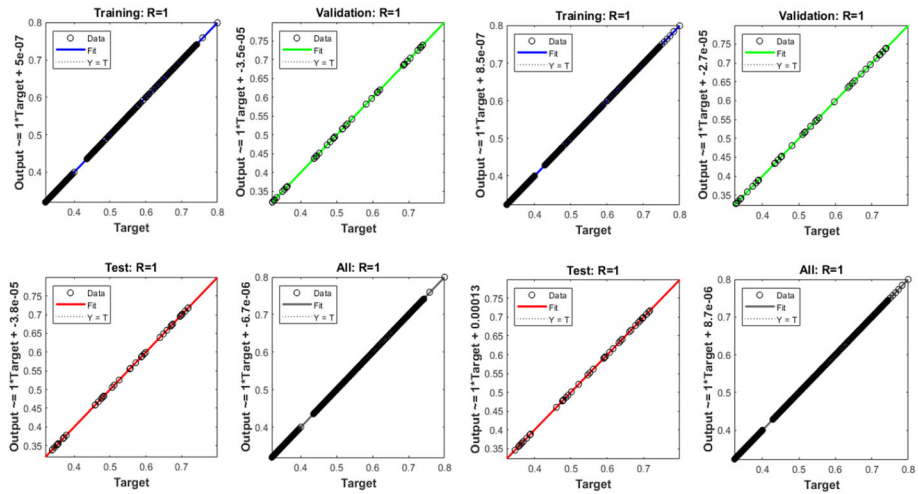


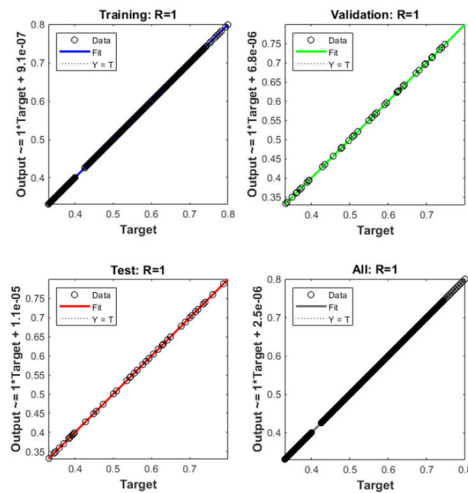
Fig. 5 Valuations and EHs to solve the FO-SEIR-KS mathematical system

certification for solving the FO-SEIR-KS nonlinear mathematical model. Thirteen numbers of neurons have been applied to solve the CVP mathematical model with KS, i.e., FO-SEIR-KS model. The numerical investigations of the FO-SEIR-KS model has been presented through the numerical stochastic procedures of the LMBS-NNs, whereas the comparative presentations have been available based on the Adams–Bashforth–Moulton. The numerical procedures of the FO-SEIR-KS system has been presented using the LMBS-NNs in order to reduce the MSE. To validate the reliability, capability and aptitude of the LMBS-NNs, the numerical results have been provided by using the STs, correlation, Ehs, regression and MSE. The matching of the obtained and reference result performances indicates the precision and accurateness of the proposed stochastic approach and the values of the AE represents the perfection of the CVP mathematical model with KS.



(a) Case 1: Regression

(b) Case 2: Regression



(c) Case 3: Regression

Fig. 6 Regression performances to solve the FO-SEIR-KS mathematical model

Table 1 Statistical values of the FO-SEIR-KS mathematical model using the LMBS-NNs

Case	M.S.E	Performance			Mu	Gradient	Epoch	Time
		[Training]	[Validation]	[Testing]				
1	1.33×10^{-08}	1.77×10^{-09}	1.98×10^{-09}	1.33×10^{-08}	1.00×10^{-09}	9.72×10^{-08}	102	3
2	7.98×10^{-09}	1.18×10^{-08}	1.24×10^{-08}	7.99×10^{-09}	1.00×10^{-09}	9.58×10^{-08}	52	2
3	1.12×10^{-10}	1.63×10^{-10}	2.33×10^{-10}	1.12×10^{-10}	1.00×10^{-10}	9.54×10^{-08}	42	2

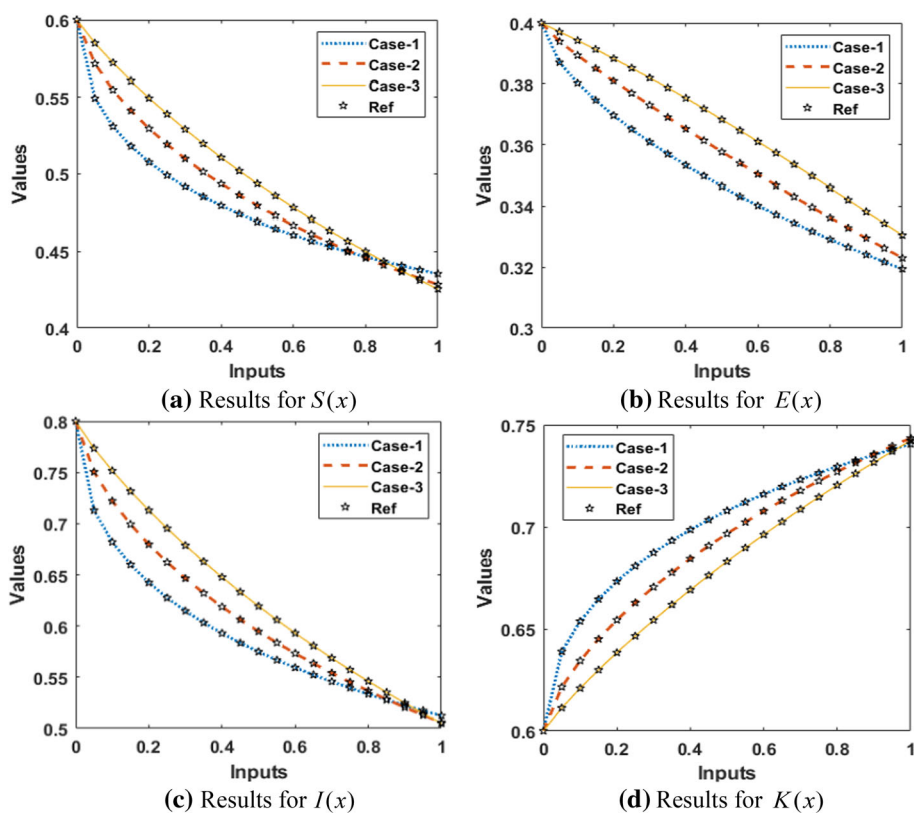


Fig. 7 Comparison of the result to solve the FO-SEIR-KS mathematical model

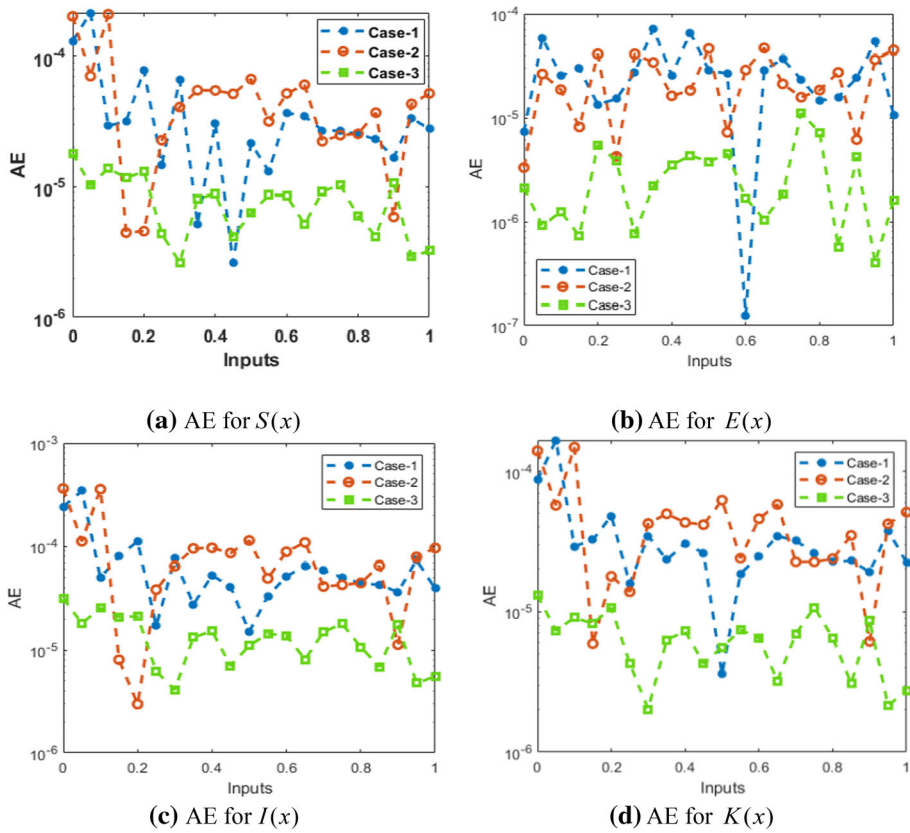


Fig. 8 AE to solve the FO-SEIR-KS mathematical model

In future, the design of the LMB-NNs procedures and their modified variants can be implemented to find the numerical solutions and analysis of the nonlinear models and systems arising in different application of utmost importance [56–63].

Acknowledgements The first author thanks the Faculty of Sciences and Liberal Arts, Rajamangala University of Technology Isan, Thailand.

Declarations

Conflict of interest All authors describe that there are no potential conflicts of interest.

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